

Modelling and validation of magnetic field distribution in permanent magnet system

Alicja Praczkowska, Michał Nowicki, Roman Szewczyk

Abstract— The modelling of the magnetic field distribution of complex systems often requires the use of complicated analytical methods. One of alternative ways of handling such an issue includes finite element method. The following paper presents that this problem may be simplified by modelling single elements and combining them into a system using Elmer software. The simulation requires the definition of element's geometry created in Netgen software, as well as the boundary conditions and the specification of material's magnetic properties. This paper discusses the whole modelling process including chart generation, results validation via measurements, and uncertainty budget.

Index Terms— Delaunay Algorithm, ElmerFEM, Finite Element Method, Magnetic Field, Magnetic Field Distribution, Magnetostatic Maxwell Equations, Netgen

1 INTRODUCTION

THE distribution of magnetic field is described by a continuous function which implies certain difficulties during its analysis. In the complex cases the analysis is conducted with the use of finite element method. The discretization of partial differential equations that define continuous physical phenomena allows to designate a finite set of points that are further utilized in computer simulations. The examined area is divided into smaller fragments which contain information about the neighboring fragments and common mesh nodes. Inside those elements the searched result of differential equations is approximated by a certain class of functions, which are possible to determine based on the values of nodes limiting the area.

The following paper presents the results of modelling of the magnetic flux density induced by ferrite permanent

experiments in order to check the correlation between the model and the real element. If the model remains consistent with the experiment, it may be used as a component of a complex system. The system that consists of validated models may then be simulated with the result in good agreement with reality.

2 MODELLING METHODOLOGY

The discussed method of the modelling of magnetic field distribution [1] utilizes Netgen 5.3 and Elmer FEM software. Both of them are free, open-source programs compatible with all popular operating systems.

The whole procedure begins from creating a text file with .geo extension, which holds the definition of element/surroundings/system geometry. It is further used to generate meshes in Netgen.

After uploading the definition of geometrical boundary conditions, the discretization of the model needs to be performed. In order to generate the mesh, Netgen uses the Delaunay algorithm [2].

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magnets. They are compared with the results of conducted

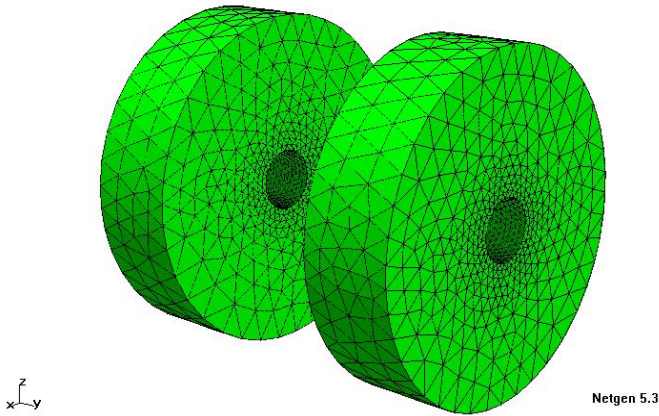


Fig. 1. The model of a two ring-magnet setup with visible mesh in Netgen 5.3 application window.

It relies on the division of the geometry into triangles under a specific condition. Circle circumscribed on any triangle cannot contain any other nodes (triangle linking points). It consists of two actions: the triangulation of boundary nodes and the generation of inner nodes. In the beginning a simple mesh is created. During the runtime, the program gradually extends it by adding further boundary nodes. After creating each node a local mesh reconstruction is performed, until it meets the algorithm’s conditions. The generation of inner nodes allows for boosting mesh quality. The software chooses the triangles that do not meet the condition of weight, equilaterally or size, and inserts a new node inside the triangle or in its neighborhood. Having done that, it conducts a local mesh recalculation in a similar way as with boundary nodes. The visible change in the size of fragments shows that the density of the mesh increases proportionally to the radius of the edge. In case of a single magnet, the number of elements was equal to 207085, while in a two magnet setup it raised to 310203.

Visible discontinuities in magnetic field distribution are caused by the precision limit in mesh setup. It should be noted that the simulation has been conducted with the precision set to ‘very fine’, which appeared to be sufficient. The further upswing of the precision causes substantial prolongation of the process, providing no significant improvement of the image. A file generated as described above is then saved with .msh extension.

The .mesh file needs to be converted in order to be compatible with ElmerFEM. The conversion is handled by ElmerGrid software. The conversion process results in creating four files: mesh.header, mesh.elements, mesh.nodes and mesh.boundary. For the calculation of the magnetic field, ElmerFEM needs two more files. One of them holds simulation parameters and setup (*.sif file). It defines used materials with their magnetic properties, which are in this case:

- air, with electric conductivity equal to 0 and a relative magnetic permeability equal to 1,
- ferrite magnet, with a relative magnetic permeability equal to 1,2 and magnetization equal to 265000 A/m along its axis.

Theoretical modelling is based on an analysis of three dimensional geometry with a finite element method. For the calculation of the magnetic field, Elmer uses magneto-static Maxwell equations[3]. They are the most basic electro-dynamical equations describing the dependencies between electric and magnetic fields, as well as the properties of magnetic field itself.

$$-\sigma \vec{E} + \nabla \times \left(\frac{1}{\mu} \vec{B} \right) = \vec{g} \tag{1}$$

The Gauss’s law[4] for magnetism in the differential form indicates that this field has zero divergence.

$$\nabla \cdot \vec{B} = 0, \quad \text{where } \vec{B} = \nabla \times \vec{A} \tag{2}$$

Another utilized Maxwell differential equation is the Faraday’s law of induction[4]. It states that in the electrical field is induced by the alternating magnetic field.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3}$$

After including boundary conditions and additional constants, as well as performing numerous transformations, Whitney edge elements method is applied[5]. It enable the program to calculate vector and scalar components which results in the complete description of magnetic field.

$$\int_{\Omega} \sigma \frac{\partial \vec{A}}{\partial t} \cdot \nabla v \, d\Omega + \int_{\Omega} \sigma \nabla V \cdot \nabla v \, d\Omega = \int_{\Omega} \nabla \cdot (\sigma \vec{E}) v \, d\Omega - \int_{\partial\Omega} (\sigma \vec{E}) \cdot \vec{n} v \, dS \tag{1}$$

$$= - \int_{\Omega} \nabla \cdot \vec{g} v \, d\Omega - \int_{\partial\Omega} (\sigma \vec{E}) \cdot \vec{n} v \, dS.$$

$$\int_{\Omega} \sigma \frac{\partial \vec{A}}{\partial t} \cdot \vec{\eta} d\Omega + \int_{\Omega} \sigma \nabla V \cdot \vec{\eta} d\Omega + \int_{\Omega} \frac{1}{\mu} (\nabla \times \vec{A}) \cdot (\nabla \times \vec{\eta}) d\Omega - \int_{\partial\Omega} \left[\left(\frac{1}{\mu} \nabla \times \vec{A} \right) \times \vec{n} \right] \cdot (\vec{n} \times \vec{\eta} \times \vec{n}) dS = \int_{\Omega} \vec{g} \cdot \vec{\eta} d\Omega,$$

In order to draw the graph of the magnetic field distribution along the z axis, a specific function is used. It saves the key values into a file as an array. As a result, the file contains 20-field array with the number of rows dependent on the number of nodes along the axis and its description. The important data is placed in columns: #5, #6 and #7, which contain the local component values of magnetic field B in x-, y- and z-axis respectively. The result needs to be ordered by the values of #19 column which holds the z-axis coordinate.

The last file needed to perform the operation of Elmer-Solver is the ELMERSOLVER_STARTINFO containing only the name of the .sif-file including extension.

The magnetic field is modelled in three-dimensional space, as shown on the figure 2. However, only the component value along the magnet axis is shown in the chapter which regards the modelling results. In case of a single magnet it is the vertical direction along z-axis. In two-magnet configuration, the y-axis component value is visualized, which is caused by the limited capabilities of utilized measurement hall sensor probes.

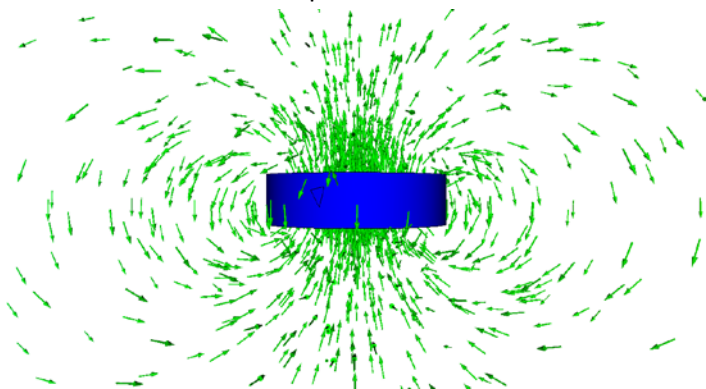


Fig.2. The resultant vectors of magnetic field. Vector length not to scale of the field value.

3 MEASUREMENT STAND

In order to verify if the theoretical model of the magnetic field is correct, a series of test has been performed

with the use of LakeShore 455 DSP Gaussmeter. Two kinds of probes have been used[6]:

- for single magnet - a probe measuring the field along the displacement direction,
- for two-magnet system - a probe measuring the field in the direction orthogonal to the displacement.

Both probes were fitted on a mount with linear millimeter scale. It allowed to move the probe vertically. Before every measurement, the devices were calibrated in dedicated ZeroGauss Chamber. Each measurement was performed in two directions along the same axis. In case of two-magnet system, a non-magnetic separator has been used. It fixed magnets' distance at 30mm.

4 MODELLING AND VALIDATION

Ferrite F30 ring magnets have been utilized for tests. The inner and outer radius were 10 and 55mm respectively, and the thickness was equal to 15mm.

Magnetic properties of the F30 material:

- remanence induction $B_r = \text{min } 0,37 \text{ T}$
- coercion $H_{cB} = \text{min. } 175 \text{ kA/m}$
- coercion $H_{cJ} = \text{min. } 180 \text{ kA/m}$
- density of magnetic energy $(BH)_{\text{max}} = 26 \text{ kJ/m}^3$

Physical properties of F30 material:

- density $\sim 4,5 \text{ g/cm}^3$
- resistivity $104 - 108 \text{ Ohm} \cdot \text{cm}$

Relative magnetic permeability for permanent (hard) magnetic materials lies between 1,1 and 1,3. For ferrite magnet it was assumed to be 1,2.

For initial assesment of ferrite magnet's magnetization, the following dependency has been used:

$$B = \mu_0(M + H) \quad \text{and for } B_r \quad H = 0$$

$$B_r = \mu_0 M$$

$$M = \frac{B_r}{\mu_0} = \frac{0,37 \text{ T}}{4 * \pi * 10^{-7} \frac{\text{H}}{\text{m}}} = 294437 \frac{\text{A}}{\text{m}} \approx 300000 \frac{\text{A}}{\text{m}}$$

It is a theoretical value, which needed to be verified by conducting appropriate laboratorial measurements. A figure presented below contains the theoretical values given by Elmer FEM software for magnetization fixed at

300 000 A/m and the respective results of measurement.

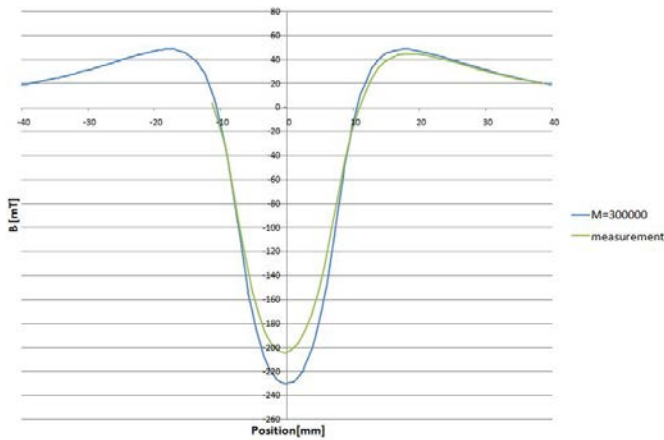


Fig.3. Value of magnetic induction B along the axis of a ring magnet. Theoretical calculation in Elmer FEM for magnetization equal to 300000 A/m and the measurement results.

As the graph shows, the assumed magnetization value is too high. For the correct calculation a proportional dependency derived from an experiment has been used. For zero displacement:

$$\frac{230 \text{ mT}}{300000 \frac{\text{A}}{\text{m}}} = \frac{203 \text{ mT}}{M}$$

$$M = \frac{203 \text{ mT}}{230 \text{ mT}} * 300000 \frac{\text{A}}{\text{m}} = 264783 \frac{\text{A}}{\text{m}} \approx 265000 \frac{\text{A}}{\text{m}}$$

A corrected value has been visualized on figure 4. along with measurements from both sides of the magnet.

In conclusion, the corrected model corresponds with reality.

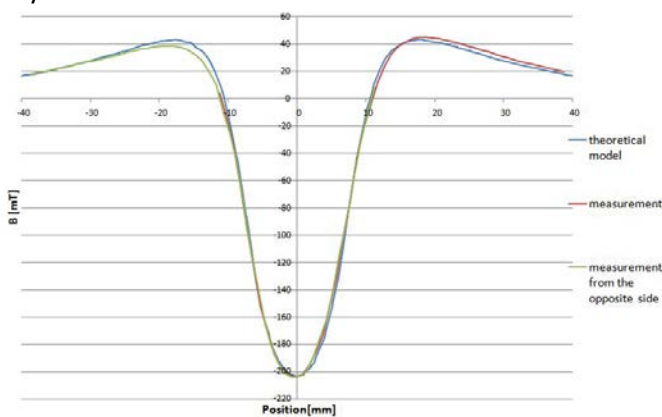


Fig.4. Absolute value of magnetic field dependent on a displacement relative from the center of ring magnet. Theoretical calculation in Elmer FEM for magnetization equal to 265000A/m and the measurement results.

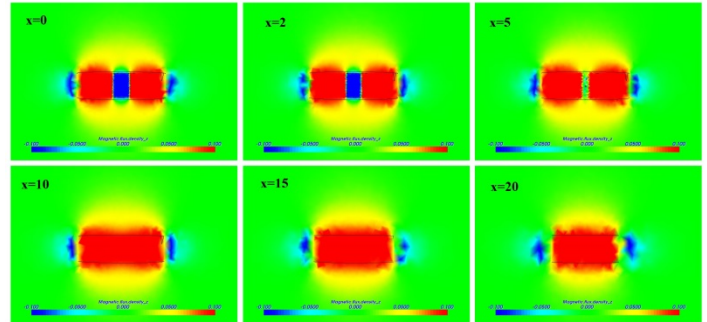


Fig.5. Clip-planes magnetic induction values distribution along the x direction (sideways regarding to magnet).

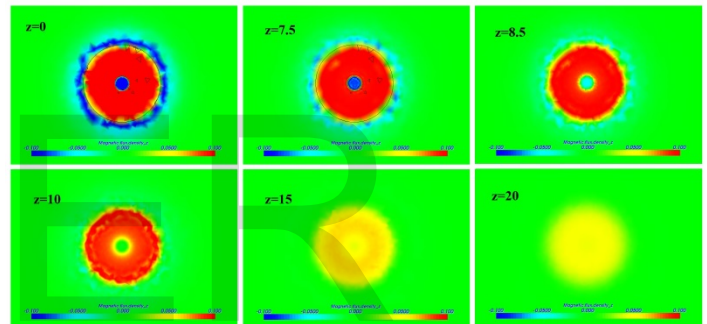


Fig.6. Clip-planes magnetic induction values distribution along the z direction (along the axis of the magnet).

A two-magnet system has been modelled in order to find the magnetic field distribution between them.

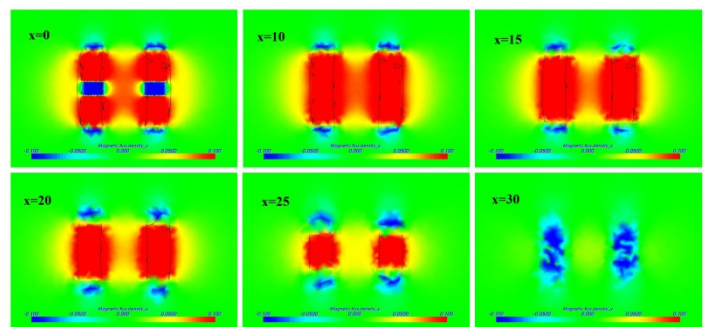


Fig.7. Clip-planes magnetic induction B values distribution along the x direction (sideways regarding to magnets).

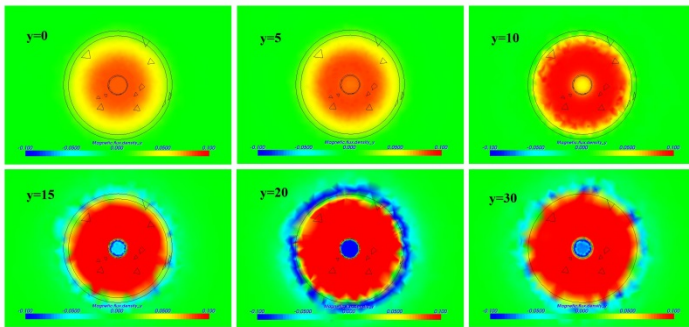


Fig.8. Clip-planes magnetic induction values B distribution along the z direction (along the axis of the magnets).

The result of the simulation has been verified by conducting an experiment similar to the one described above. Both theoretical and experimental results have been placed on a common graph, shown in figure 9.

The conclusion is that the prepared modelling algorithm corresponds with reality and may be further used to designate the magnetic field distribution of complex systems.

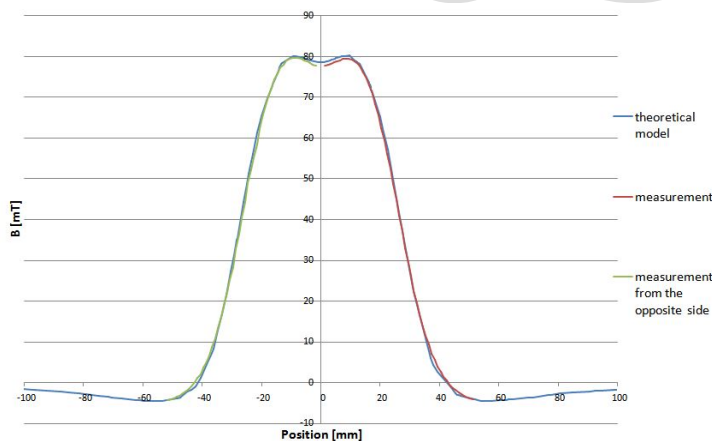


Fig.9. Value of magnetic field along the axis between the ring magnets of a two-magnet system. Theoretical calculation in Elmer FEM for magnetization equal to 265000A/m and the corresponding real-world measurement results.

5 MEASUREMENT UNCERTAINTY

Before conducting an experiment a background field has been measured, in order to eliminate environment error influence on the obtained results. While descending the probe, a gradual increase of the indicated value could be observed. The reason of such behavior lies in interaction with utilized equipment and random noise in the laboratory. In the lowest position its value reached 0,04-0,05 mT so it may be omitted as it is far smaller than the magnets magnetization.

More significant was the error coming from probe positioning against the magnets. Considering all uncertainties of the positioning of magnets and probe in the horizontal direction, the maximal difference in indications did not exceed 0,05mT.

Another component of the total uncertainty is the vertical displacement which changes a result by approximately 0,9mT.

The last factor contributing to uncertainty budget is the limited measurement precision of the probe itself. The same applies to gaussmeter. The total uncertainty coming from these devices was estimated as 0,01mT.

In the end, the total uncertainty of the measurement process appears to be more than sufficient to designate the procedure as very precise. This is further proved by the high correlation of the results of both theoretical and experimental approach of finding single magnet's magnetic field.

6 CONCLUSION

The results of the experiment have high agreement with the theoretical model received by the simulation. It proves the correctness of performed calculations as well as the genuineness of the examined magnetic phenomena. During the design of the device that uses permanent magnet system, a single element ought to be modelled and tested. It is essential to ensure that the parameters chosen for the simulation are correct. This ensures the correctness of the modelled three dimensional magnetic field distribution of the more complicated system. Particularly worth recommendation is the ElmerFEM software, as it allows to

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calculate the effect of various phenomena by using finite element method, including magnetostatics and magneto-dynamics. Other applications are the calculations of fluid flows, stress distribution or heat propagation. It is also possible to develop solvers that are based on custom derived equations. All of the described programs are free and available under an open-source license.

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